APPLIED

Magnetics

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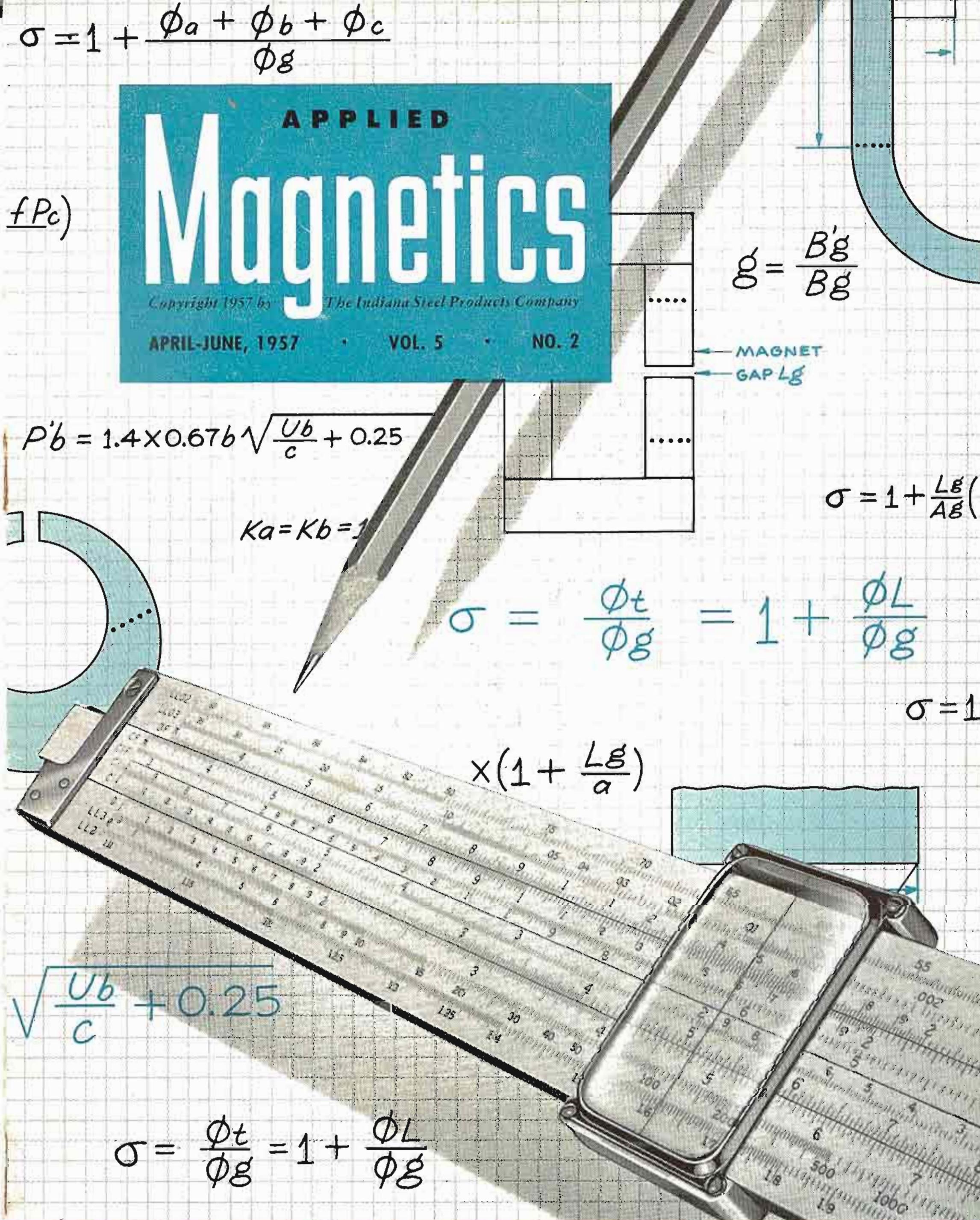

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- Indiana Magnetizers for On-the-Spot Magnetizing

Published by the makers of INDIANA PERMANENT MAGNETS

ESTIMATING LEAKAGE FACTORS FOR MAGNETIC CIRCUITS BY A SIMPLE METHOD

By R. K. Tenzer, Scientist
The Indiana Steel Products Company

Reprinted from *Electrical Manufacturing*, February, 1957

The exact calculation of magnetic flux through air is possible only for some ideal arrangements (e.g., flux distribution between two parallel cylinders of infinite length) that are not dealt with in actual practice. The design engineer, therefore, depends entirely on approximations when calculating leakage factors for magnetic circuits.

With drastic simplifications of the leakage flux paths, permeances, and magnetomotive forces, the leakage factor can be calculated by a single formula that contains only constants and geometric dimensions. The magnetic circuits to be considered here are those where one or more permanent magnets and soft steel parts are placed symmetrically in relation to an air gap. All circuit elements operate below saturation and are arranged in series. Three basic arrangements are shown in Figs. 1-3.

Work was sponsored by Wright Air Development Center, USAF.

NOTE: *For Electromagnetic Circuits operating in the unsaturated state, the procedure outlined in this article for permanent magnets also applies; the coil-covered parts correspond to the permanent magnet parts.*

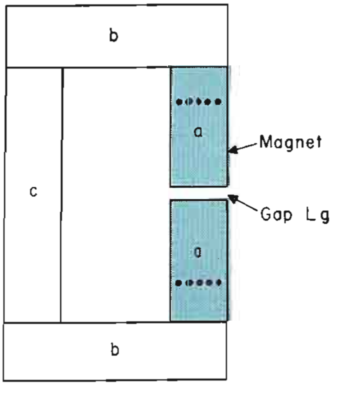

Fig. 1

LEAKAGE FLUX AND LEAKAGE FACTOR

Due to magnetic leakage only a part ϕ_g of the total flux ϕ_t through the neutral zone(s) of the permanent magnet(s) is found in the air gap. The difference between these two values is known as leakage flux.

$$\phi_L = \phi_t - \phi_g \tag{1}$$

In practical magnetic design, leakage is considered in terms of a leakage factor

$$\sigma = \frac{\phi_t}{\phi_g} = 1 + \frac{\phi_L}{\phi_g}. \tag{2}$$

For the purpose of simplification the leakage flux can be assumed to follow three basic, probable paths: ϕ_a between parts *a*; ϕ_b between parts *b*; and ϕ_c along part *c*. (See Fig. 3.) Equation (2) then becomes

$$\sigma = 1 + \frac{\phi_a + \phi_b + \phi_c}{\phi_g}. \tag{3}$$

By using the relation $\phi = \text{mmf} \times P$, where mmf stands for magnetomotive force and P for permeance, Eq. (3) becomes

$$\sigma = 1 + \frac{\text{mmf}_a P_a + \text{mmf}_b P_b + \text{mmf}_c P_c}{\text{mmf}_g P_g},$$

which, for expediency, can be written

$$\sigma = 1 + \frac{1}{P_g}\left(\frac{\text{mmf}_a}{\text{mmf}_g} P_a + \frac{\text{mmf}_b}{\text{mmf}_g} P_b + \frac{\text{mmf}_c}{\text{mmf}_g} P_c\right). \tag{4}$$

When the mmf ratios are denoted by the letter K, Eq. (4) becomes

$$\sigma = 1 + \frac{1}{P_g}(K_a P_a + K_b P_b + K_c P_c). \tag{5}$$

This equation serves as a basis for numerical calculations of leakage factors and can be converted to a relation containing only geometric dimensions and some constants.

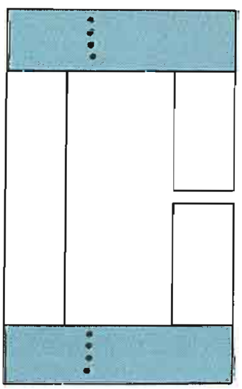

Fig. 2

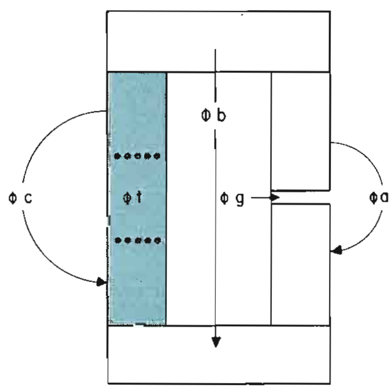

Fig. 3

SIMPLIFIED LEAKAGE PERMEANCES

For the calculations of leakage permeances between soft steel parts, the following simplified relations have proved to be satisfactory:

$$P_a = 1.7 \times U_a \times \frac{a}{a + L_g} \tag{6}$$

$$P_b = 1.4 \times b \times \sqrt{\frac{U_b}{c} + 0.25}; \text{ where } 0.25 \leqq \frac{U_b}{c} \leqq 4. \tag{7}$$

Here U stands for the cross-section perimeter of the parts as defined in Fig. 1. Equation (6) is based on a derivation by T. van Urk. (*1*)† Equation (7) is a simplification of a rather complicated relation obtained from potential theory. (*2*) The total length of part b is used although the surface portions adjacent to parts a and c do not contribute to leakage. The leakage flux emanating from the end faces of parts b is assumed to balance this lack.

The fact that permanent magnets have a neutral zone which does not contribute to leakage is taken into account by using only two-thirds of the magnets' total length when calculating their leakage permeances (P'). Actually the length of the neutral zone varies as the air gap is varied. However, it appears to be a satisfactory approximation to always use two-thirds of a magnet's length as "effective length."

With $a' = 0.67a$ and $b' = 0.67b$

$$P'_a = 1.7\ U_a \frac{0.67a}{0.67a + L_g} \tag{6a}$$

$$P'_b = 1.4 \times 0.67b \sqrt{\frac{U_b}{c} + 0.25} = 0.67\ P_b. \tag{7a}$$

When part c consists of permanent magnet material its permeance can be calculated as

$$P_c = 0.5\ U_c. \tag{8}$$

This formula is a crude simplification obtained by applying Eq. (6) to part c, considering the neutral zone $c/3$ as an air gap, and taking the different distributions of leakage flux density into account.

The permeance of the air gap is simply written as

$$P_g = A_g/L_g. \tag{9}$$

SIMPLIFIED MMF RATIOS

Simple expressions for the mmf ratios are obtained by the following approximations:

1. The reluctance in *soft steel parts* is neglected. Hence

$$\text{mmf}_a = \text{mmf}_b = \text{mmf}_g \text{ or}$$
$$K_a = K_b = 1$$
$$(\text{mmf}_c = 0 \text{ and therefore } K_c = 0). \tag{10}$$

2. Since the mmf along *permanent magnet parts* is not constant, mean integral values $(\overline{\text{mmf}})$ are used. Experimental data suggest that two-thirds of $\text{mm}f_g$ is the "effective mmf" for leakage flux between permanent magnet parts, hence

$$\overline{\text{mmf}_a} = \overline{\text{mmf}_b} = \overline{\text{mmf}_c} = 2/3\ \text{mmf}_g \text{ or}$$
$$K_a = K_b = K_c = 2/3. \tag{11}$$

To establish this approximation, the distributions of leakage flux along permanent magnets and soft steel in the same location of the magnetic circuit were measured. For soft steel in parts a the leakage flux density increased

†Italic numerals in parentheses refer to Cited References at end of article.

COVER STORY

A single formula for leakage factors that contains only geometric dimensions and constants can be adapted to various magnetic circuits. By using this method the design engineer can reduce time required to calculate leakage factors to as much as one fourth in many practical applications.

toward the air gap roughly according to a linear function. A square function was obtained for permanent magnets in parts *a*. Leakage flux density along parts *b* was almost constant for soft steel. For permanent magnets in part *b* or *c* the distribution of leakage flux density could be approximated by a square root function.

These functions have been used to calculate mean integral values for the different distributions of leakage flux density. The ratio of the mean values for permanent magnets to those of soft steel in parts *a* was 2/3. The same value was obtained for parts *b*. We introduced this factor also to part *c* for the matter of consistency. The corresponding mmf ratios have the same value because outside the neutral zone permanent magnets are taken to have the same leakage permeance as soft steel parts of equal dimensions.

BASIC FORMULAS

Inserting the permeances for soft steel into Eq. (5) leads to the general formula

$$\sigma = 1 + \frac{L_g}{A_g}\left(K_a \times 1.7U_a \frac{a}{a + L_g} + K_b \times 1.4b \sqrt{\frac{U_b}{c} + 0.25} + K_c \times 0.5\, U_c\right). \quad (12)$$

This formula contains only constants and geometric dimensions. It can easily be modified to fit each of the three basic arrangements of Figs. 1-3 through two simple rules based on the "effective length" of each part and the "effective mmf" of each flux path:

1. For a leakage flux path between soft steel parts, use their total lengths and a constant *K* of 1.

2. For a leakage flux path between permanent magnet parts, use two-thirds of their total lengths and a constant *K* of 0.67.

Thus Eq. (12) can be modified to the following expressions for the leakage factors of the basic arrangements of Figs. 1-3.

Fig. 1: $$\sigma = 1 + \frac{L_g}{A_g} \times 0.67 \times 1.7\, U_a \frac{0.67a}{0.67a + L_g} \quad (13\text{-}1)$$

Fig. 2: $$\sigma = 1 + \frac{L_g}{A_g}\left(1.7\, U_a \frac{a}{a + L_g} + 0.67 \times 0.67 \times 1.4b \sqrt{\frac{U_b}{c} + 0.25}\right) \quad (13\text{-}2)$$

Fig. 3: $$\sigma = 1 + \frac{L_g}{A_g}\left(1.7\, U_a \frac{a}{a + L_g} + 1.4b \sqrt{\frac{U_b}{c} + 0.25} + 0.67 \times 0.5\, U_c\right) \quad (13\text{-}3)$$

Equation (13-1) takes into account only leakage flux between the two permanent magnets. However, there is also leakage between each magnet and the adjacent soft steel parts in Fig. 1. This leakage becomes greater as the gap length is increased. It cannot be neglected when the second addend in Eq. (13-1) is greater than 1. To avoid another leakage flux path in these simple calculations requires that this additional leakage flux be considered by means of a corrective factor. Equation (13-1) then becomes

$$\sigma = 1 + \frac{L_g}{A_g} \times 0.67 \times 1.7\, U_a \frac{0.67a}{0.67a + L_g} \times \left(1 + \frac{L_g}{a}\right). \quad (13'\text{-}1)$$

In Fig. 2 there is leakage flux along each permanent magnet, which is not considered in Eq. (13-2). The permeance of these two flux paths is similar to Eq. (8). The mmf of each path is approximately one-third of mmf_g. Equation (13-2) can be corrected by adding $2 \times .33 \times .5\ U_b$ in the parentheses:

$$\sigma = 1 + \frac{L_g}{A_g}\left[1.7\ U_a \frac{a}{a + L_g} + 0.67 \times \left(0.67 \times 1.4b \sqrt{\frac{U_b}{c}} + 0.25 + 0.5\ U_b\right)\right]. \quad (13'\text{-}2)$$

In most cases this addend is small compared to the other addends within the brackets. For this reason it can quite often be neglected, as is done in the text that follows.

TYPICAL APPLICATIONS

The validity of Eq. (12) can be extended to magnetic circuits which resemble the three basic arrangements. If, for example, two soft steel pole pieces (of length e each, see Fig. 4) are attached to the magnets in arrangement of Fig. 1, the leakage flux ϕ_a can be divided into two portions: leakage flux between the soft steel pole pieces and leakage flux between the magnets. For the second portion $L_g + 2e$ is considered the length of the air gap, and Eq. (13′-1) becomes

$$\sigma = 1 + \frac{L_g}{A_g} \times 1.7U_a \left[\frac{e}{e + L_g} + 0.67 \frac{0.67a}{0.67 + L_g + 2e} \times \left(1 + \frac{L_g + 2e}{a}\right)\right]. \quad (14)$$

For tapered pole pieces (frustrum shaped, of length e each, see Fig. 5) only a slight change has to be made in Eq. (14) by using the area of the frustrum face, A'_g, as gap area:

$$\sigma = 1 + \frac{L_g}{A'_g} \times 1.7\ U_a \left[\frac{e}{e + L_g} + 0.67 \frac{0.67a}{0.67a + L_g + 2e} \times \left(1 + \frac{L_g + 2e}{a}\right)\right]. \quad (15)$$

The derivation of Eq. (15) from Eq. (14) is based on the assumption that the total leakage flux emanating from the curved surfaces of conic pole pieces and from their fringes is approximately equal to the leakage flux emanating

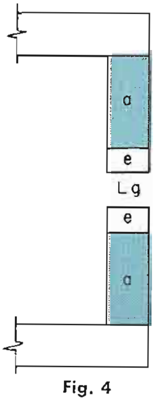

Fig. 4

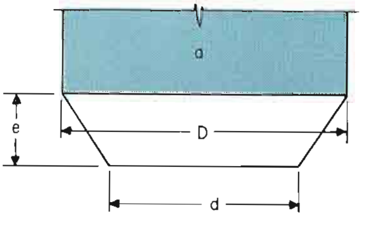

Fig. 5

from the curved surfaces of cylindrical pole pieces (of the same axial length) and their fringes. This assumption is reasonably fulfilled when $d > 2e$ and $L_g > e$. A comparison of calculated with measured values indicated that good results can be obtained for half-angles of the cone varying between 40 and 55 deg.

Gain Factor. The similarity of Eqs. (14) and (15) makes it possible to calculate the gain of flux density in the air gap obtained by applying conic instead of cylindrical pole pieces while maintaining the length of the gap. A gain factor can be defined by the ratio of the gap flux density with conic (B'_g) and cylindrical pole pieces (B_g)

$$g = \frac{B'_g}{B_g}. \tag{16}$$

By priming all quantities which refer to the application of conic pole pieces, a short calculation leads to

$$g = \frac{(1 + L_g/A_g \times P_L)\, A_g}{(1 + L_g/A'_g \times P_L)\, A'_g} \times \frac{B'_d}{B_d}. \tag{17}$$

Here P_L stands for the total leakage permeance and B_d for the remanent induction in the magnet. When B_d and the respective permeance (load line) coefficient p ($p = B_d/H_d$ on demagnetization curve) are given, B'_d is found calculating $p' = (P'_L + P'_g) \times p / (P_L + P_g)$ and looking up the corresponding B'_d value on the demagnetization curve of the material.

According to Eq. (17) there is no considerable gain where $P_L \times L_g A'_g >> 1$. There may be even a loss in gap flux density ($g < 1$), when in addition, B'_d deviates drastically from B_d.

Equation (17) may be generalized to allow the calculation of gain or loss in gap flux density due to a varied gap length L'_g, gap area A'_g or leakage permeance P'_L. The gain factor then becomes

$$g = \frac{(1 + L_g/A_g \times P_L)\, A_g}{(1 + L_g/A'_g \times P'_L)\, A'_g} \times \frac{B'_d}{B_d}. \tag{18}$$

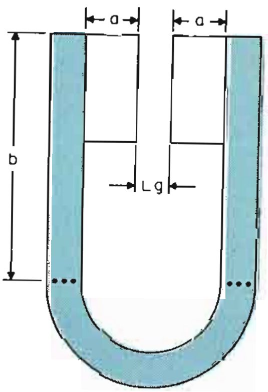

Fig. 6

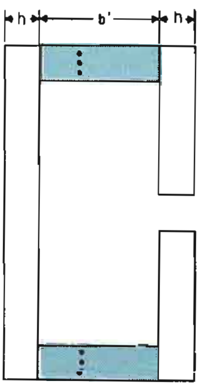

Fig. 7

Other Variations. Three more examples of the application of Eq. (12) to modified magnetic circuits are shown in Figs. 6-8.

In an arrangement consisting of a U-shaped magnet with soft steel pole pieces and air gap (see Fig. 6) the curved portion can be considered the neutral zone. Consequently, the total length b of the straight portions contributes to leakage. With $K_b = 0.67$

$$\sigma = 1 + \frac{L_g}{A_g}\left(1.7\ U_a \frac{a}{a + L_g} + 0.67 \times 1.4b\sqrt{\frac{U_b}{c} + 0.25}\right). \tag{19}$$

In Fig. 7, ϕ_b is divided into two portions and Eq. (13-2) becomes

$$\sigma = 1 + \frac{L_g}{A_g}\left[1.7\ U_a \frac{a}{a + L_g} + \left(1.4h + 0.67^2 \times 1.4b'\right)\sqrt{\frac{U_b'}{c} + 0.25}\right]. \tag{20}$$

For ring magnets (see Fig. 8), the portions which contribute to leakage are treated as if they were straight parts. They correspond to parts a in Fig. 1. Length 'a' is calculated as one-third of the average length of the ring magnet.

$$a = \frac{\pi}{6}\ (D + d - 2\ L_g). \tag{21}$$

where D and d are outer and inner diameters. The perimeter of the gap is arbitrarily taken as U_a; this also holds for eccentric arrangements. The leakage factor then becomes

$$\sigma = 1 + \frac{L_g}{A_g} \times 0.67 \times 1.7\ U_a \frac{0.67a}{0.67a + L_g} \times \left(1 + \frac{L_g}{a}\right). \tag{22}$$

The accuracy of leakage factors calculated according to the given rules was checked by comparison with measured values on many different magnetic circuits. Deviations were less than ± 10 per cent. O O O

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1. "Use of Modern Steels for Permanent Magnets," A. T. van Urk, *Phillips Technical Review,* Vol. 5, 1940, pp. 29-35.
2. "Electromagnetic Devices," H. C. Rotors, J. Wiley & Sons, Inc., New York, 1941, p. 119.

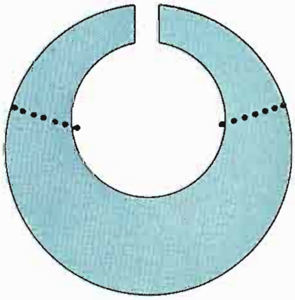


Fig. 8

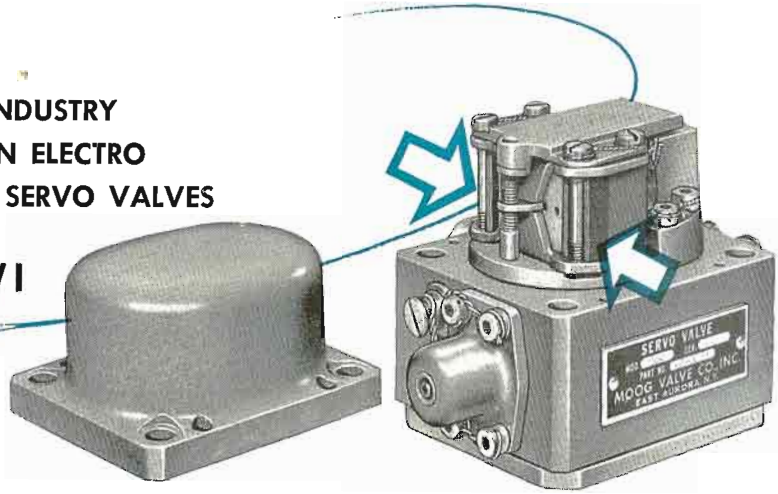
EDITOR'S NOTE:

Additional copies of this issue, containing this time saving and simplified method to calculate leakage flux, may be obtained by writing on your company letterhead to the Editor.

AIRCRAFT INDUSTRY DEPENDS ON ELECTRO HYDRAULIC SERVO VALVES

Using

ALNICO VI



Two, small Indiana Alnico VI magnets (arrows) are the heart of the first stage, or hydraulic amplifier, of Moog Valve Company's servo valve. This type of valve is now specified for over half of the fighter-interceptor aircraft and the majority of operational missiles.

Because of their application on aircraft and missiles, valve weight and space requirements must be kept to a minimum. These valves depend on the product stability characteristics of Indiana's permanent magnets to insure reliability of performance.

Use of the Moog double-nozzle-flapper design principle in electro-hydraulic components has resulted in servo controls of high speed and extreme accuracy.

In addition to the torque-motor magnets found in all Moog valves, earlier valves (still in wide use) employed two small magnetic traps for the purpose of removing magnetic contamination from the user's hydraulic system. This offers protection to the oil-immersed first stage air-gap and increases valve life. Again Indiana's Alnico VI does the required job.

Behind the Scenes

Meet Charles H. Repenn . . .

. . . New Sales Engineer in Indiana Steel's Cleveland Sales Office, replacing Stuart C. Beyerl, who will be moving to the company's New York Sales Office. Some readers in the Chicago area may have already met "Chuck", who has been calling on customers in that area since completing an intensive training program in Valparaiso covering permanent magnet design, manufacture and materials. Mr. Repenn has had previous sales engineering experience with the Minnesota Mining & Manufacturing Company before joining Indiana Steel Products. He has his degree in Electrical Engineering from the University of Illinois.

NEW INDOX V CERAMIC MAGNETS

3½ TIMES STRONGER THAN CONVENTIONAL CERAMIC MAGNETS

IDEAL FOR:

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- **Synchronous drives**
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- **Eddy current drives**
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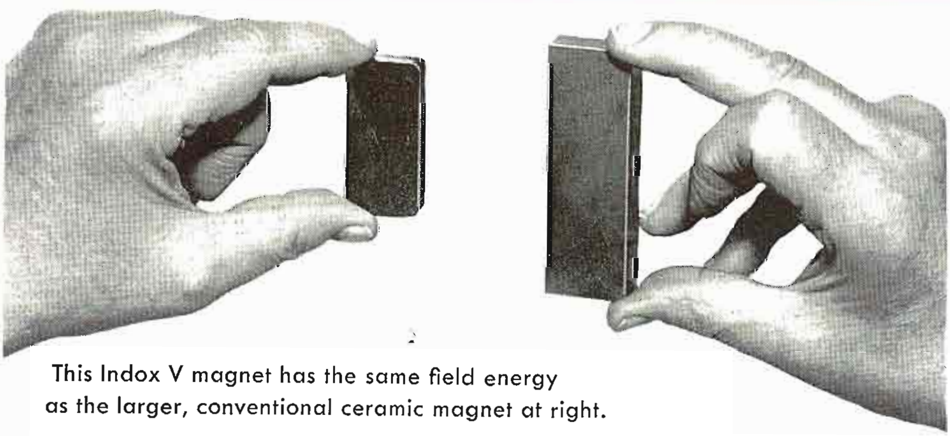
Indox V—another first from the research and development laboratories of The Indiana Steel Products Company—is available to magnet users immediately. This unique, new magnet requires no critical materials. It is a highly oriented barium ferrite using inexpensive, non-critical, raw materials that are constantly available. Shortages in times of emergency cannot occur.

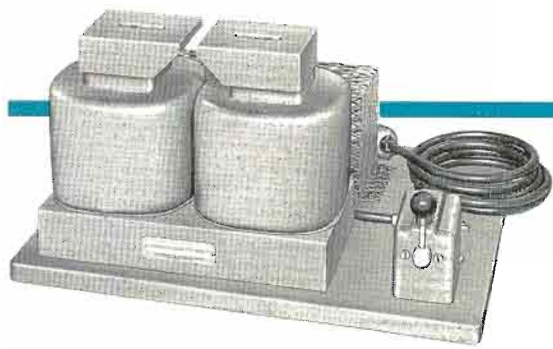
Another advantage of Indox V is it requires less space, weight to do same job. Volume and weight comparisons show that the energy of Indox V far exceeds Indox I and is comparable to Alnico V, the strongest permanent magnet material commercially available.

Indox V also offers high resistance to demagnetization. The material can be designed for applications where extremely high demagnetizing forces exist . . . without irreversible losses occuring. This means it can be used where other types of magnets have been impractical . . . for example, in stators of medium-size electric motors where electromagnets are now being used. Medium-size D.C. motors using Indox V have consistently exceeded specifications in tests and may replace the wound field type on an economic basis.

For complete specifications, write for engineering Bulletin No. 16-F.



This Indox V magnet has the same field energy as the larger, conventional ceramic magnet at right.



INDIANA MAGNETIZERS

for "on-the-spot" magnetizing of permanent magnets

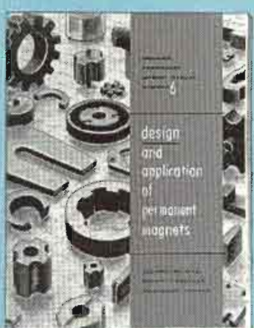
Many advantages are offered with an Indiana magnetizer. You can get optimum magnetic results where the magnet can be magnetized after assembly. Loss of magnetization due to mishandling is reduced. Contamination due to pick up of magnetic particles is reduced. There is no need to buy magnets with special packaging or keepering.

A saving in time and costs is realized since magnets need not be returned to the manufacturer for magnetizing.

Indiana magnetizers are available in either the electromagnetic or the permanent magnet type. Either one of these types will meet the requirements of most applications.

Indiana magnetizers are easy to use. Merely set the air gap adjustment and place the magnet in the proper position. With Indiana's electromagnetic type, the switch is turned on for a second or two. In using the permanent magnet magnetizers the magnets are placed in contact with the pole pieces and removed immediately.

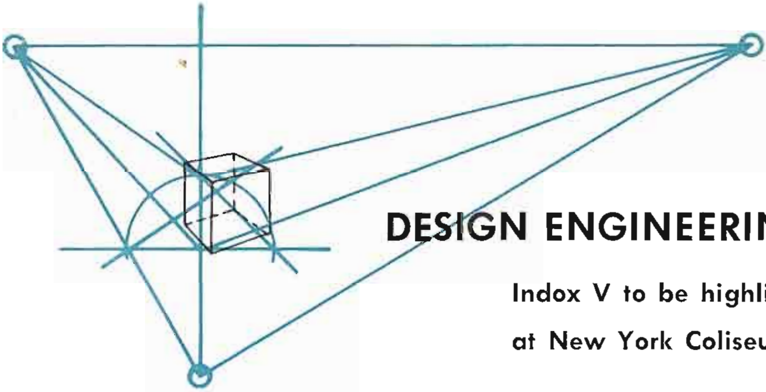
Complete specifications may be obtained by writing for bulletin No. 17-F. Indiana's Engineering Division can assist in the selection of a magnetizer appropriate to particular requirements.



NEW DESIGN MANUAL NO. 6

This important reference book for design engineers, "DESIGN AND APPLICATION OF PERMANENT MAGNETS," will be available in the near future. In addition to discussion on the functions and application of permanent magnets, Manual No. 6 will contain chapters on magnetic circuits, fixed and variable air-gaps, external demagnetization influences, and many other important aspects of magnet design.

Copies will be distributed without charge by writing The Indiana Steel Products Company, Dept. DM, Valparaiso, Indiana.



DESIGN ENGINEERING SHOW

Indox V to be highlighted at New York Coliseum

The second Design Engineering Show will be held at the New York Coliseum, May 20-23, 1957.

The conference in conjunction with the show is sponsored by the Machine Design division, American Society of Mechanical Engineers. Basic elements of product design, such as mechanical, materials and electrical, will be dealt with on each day in separate conference sessions, enabling engineers to devote their conference time most profitably to the problems that concern them most.

Thousands of design engineers will have an opportunity to examine Indiana's new, high-energy Indox V and its applications (for a review of Indox V, see page 10).

Samples and specifications on Indiana's complete line of permanent magnet materials will also be available. All shapes, all sizes, all grades, Cast Alnico, Sintered Alnico, Hyflux Alnico, Ceramic Indox, Cunife and other special magnet materials will be exhibited.

Members of the design staff and sales engineers will be on hand ready to cope with various problems. Whatever the design engineer needs, Indiana can recommend the material that's best for the individual product.

Be sure to visit Indiana's exhibit. The booth number is 622.

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